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A PROBABILISTIC INTERPRETATION OF THE FRACTIONAL-ORDER DIFFERENTIATION

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Abstract

The theory of Fractional Calculus (FC) is a useful mathematical tool for applied sciences. Nevertheless, FC is somehow hard to tackle and only in the last decades researchers were motivated for the application of the associated concepts. There are several reasons for this state of affairs, namely the apparent 'sufficiency' of classical differential calculus for real-world applications, the plethora of different definitions for fractional derivatives and integrals and the lack of a simple interpretation for such formulae. In what concerns the FC usefulness in the case of physics and engineering sciences, the progress in the areas of chaos and fractals lead to the development of fractional-order models and algorithms. On the other hand, the conceptual analysis of a fractional integral or a fractional derivative has also been addressed, but a simple interpretation is not yet completely established.

This paper discusses a probabilistic interpretation of the fractional-order derivative, based on the Grünwald-Letnikov definition, that reduces to the standard geometric interpretation for the limit cases of integer order, namely for the derivatives of order one and zero.

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